

An FPGA based sensor fusion algorithm for IMU data processing

Pietro Pennestrì

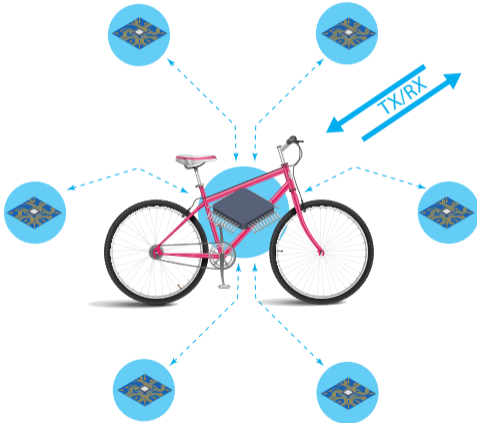
Agenda

- Project Background and Introduction
- Sensor Fusion
 - Introduction to IMU sensor Fusion
 - Proposed sensor fusion algorithm
 - Proposed FPGA architecture
- Approximation of ISQRT and SQRT
 - Polynomial approximation
 - Proposed FPGA architecture
- Conclusion and Future Work

Project Background

- The bike is recognized as a sustainable and energy-efficient method of transport.
- Although mainly limited to electric motor power control and to aftermarket accessories, there is a generalized trend toward bikes digitalization.
- The growing trend is witnessed by the wide use in commercial advertisement of the term **Smart Bike**.

Smart Bike Proposed Definition



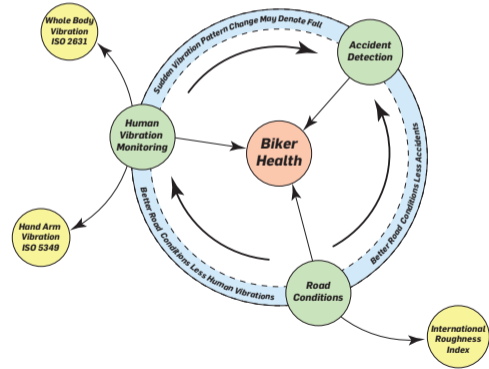
Smart Bike

A **SMART BIKE** is a bike equipped with a variety of modular sensors & devices, connected to an onboard master unit, also capable of **data processing** and **transmission**, for the purpose of **enriching the user experience**, increasing his/her **awareness** and **safety**.

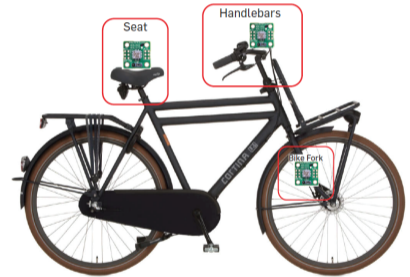
V-AUDIT: a vibration monitoring device

V-AUDIT stands for **V**ibration **A**udit and is herein proposed as a low cost bike accessory composed of:

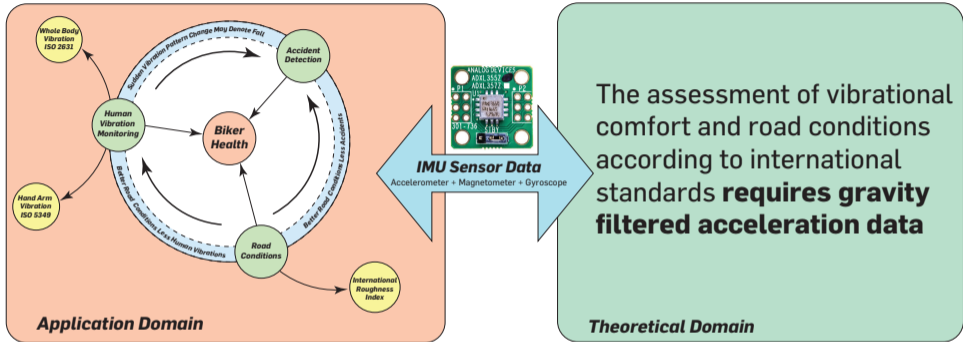
- a unit to collect and onboard process different Inertial Measurement Unit (IMU) sensors data
- different IMUs positioned at bike-human body interfaces (seat, handlebar).



- The sensors on board 9-DoF IMU provide all data for the functional features of V-AUDIT. ✓
- Low Cost and easily installed on bikes. ✓
- Operations required: filtering and calibration. ✗

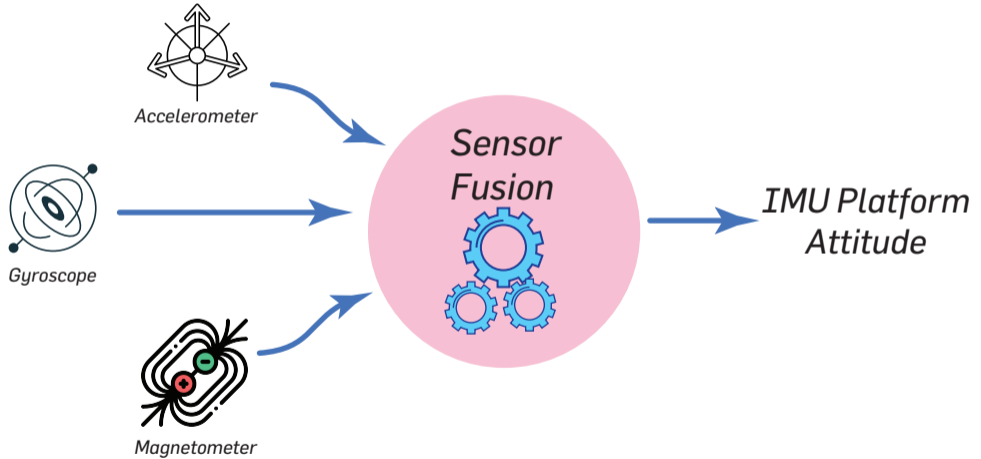


Research Focus

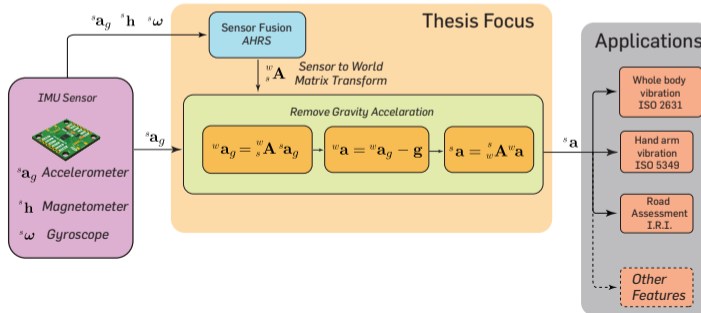
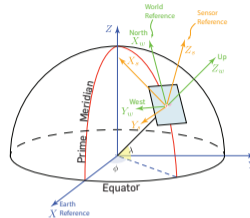


How to remove gravity from IMU accelerometer data ? How to implement it into FPGA?

Sensor Fusion



Application of Sensor Fusion results



Vector Sensor to World Coordinate Frame Transform

Quaternion

Let the quaternion

$$\underline{\mathbf{q}} = \underline{\mathbf{e}} = e_0 + e_1 \vec{i} + e_2 \vec{j} + e_3 \vec{k} \text{ then:}$$

$${}^w \underline{\mathbf{v}} = {}^w \underline{\mathbf{q}} \otimes {}^s \underline{\mathbf{v}} \otimes {}^w \underline{\mathbf{q}}^*$$

Rule for quaternion multiplication:

$$\begin{aligned} \underline{\mathbf{q}}_b \otimes \underline{\mathbf{q}}_a = & (q_{0a}q_{0b} - q_{1a}q_{1b} - q_{2a}q_{2b} - q_{3a}q_{3b}) + \\ & + (q_{0a}q_{1b} + q_{1a}q_{0b} + q_{2a}q_{3b} - q_{3a}q_{2b}) \cdot \vec{i} + \\ & + (q_{0a}q_{2b} - q_{1a}q_{3b} + q_{2a}q_{0a} + q_{3a}q_{1b}) \cdot \vec{j} + \\ & + (q_{0a}q_{3b} + q_{1a}q_{2b} - q_{2a}q_{1b} + q_{3a}q_{0b}) \cdot \vec{k} \end{aligned}$$

Matrix

$$\mathbf{E} = \begin{bmatrix} -e_1 & e_0 & -e_3 & e_2 \\ -e_2 & e_3 & e_0 & -e_1 \\ -e_3 & -e_2 & e_1 & e_0 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} -e_1 & e_0 & e_3 & -e_2 \\ -e_2 & -e_3 & e_0 & e_1 \\ -e_3 & e_2 & -e_1 & e_0 \end{bmatrix}$$

$${}^w \mathbf{A} = \mathbf{E} \mathbf{G}^T$$

$${}^w \underline{\mathbf{v}} = {}^w \mathbf{A} {}^s \underline{\mathbf{v}}$$

Quaternions .vs. Matrices

	Quaternions	Matrices
Easiness of Manipulation	✓	✓
Computational Efficiency	✗	✓
Conciseness in Notation	✓	✗
Interpolation	✓	✗
Numerical Stability	✓	✗

Quaternions Properties

Quaternion components normalization property

$$\mathbf{q}^T \mathbf{q} = e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$$

Quaternion consistency condition

$$E_m = \dot{\mathbf{q}}^T \mathbf{q} = 0$$

Objective function to minimize

$$F = E_m^2$$

Sensor Fusion of IMU Data

- Due to its importance and pervasive applications, Sensor Fusion is a commonly required task in IoT devices.
- For IMU sensor fusion, those of Mahony and Madgwick provided the theoretical ground work for many algorithms.
- None of them include in the computational flow the a consistency condition between quaternion and their time derivative.

Proposed Improvements of Madgwick Algorithm

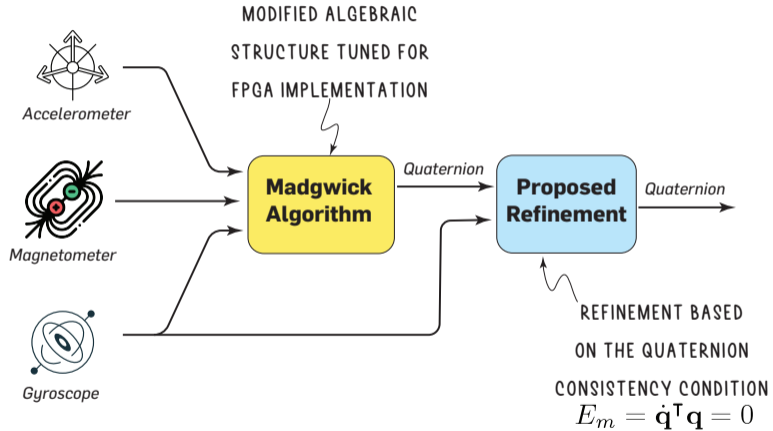
- Translation of the Madgwick algorithm into matrix notation.
Effect: Execution time reduced, easy operations scheduling.

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Proposed Improvements of Madgwick Algorithm

- Translation of the Madgwick algorithm into matrix notation.
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- Simplification of the gradient algebraic structure.
Effect: Execution time reduced, easy operations scheduling.
- Addition of a quaternion consistency condition.
Effect: Accuracy increase. A norm is introduced to evaluate IMU sensor fusion algorithms.



The Modified Madgwick Algorithm

- **Data:** ${}^s\mathbf{a}_{i+1}$, ${}^s\mathbf{h}_{i+1}$, ${}^s\boldsymbol{\omega}_{i+1}$, \mathbf{q}_i
- **Constants:**
 ${}^w\mathbf{g} = [0 \ 0 \ -1]^T$, β , Δt , a_{th} , n
- **if** $\|{}^s\mathbf{a}_{i+1}\| \leq a_{th}$ **then** $a = \text{TRUE}$ **else** $a = \text{FALSE}$
- **Normalize:** ${}^s\mathbf{a}_{i+1} \leftarrow \frac{{}^s\mathbf{a}_{i+1}}{\|{}^s\mathbf{a}_{i+1}\|}$, ${}^s\mathbf{h}_{i+1} \leftarrow \frac{{}^s\mathbf{h}_{i+1}}{\|{}^s\mathbf{h}_{i+1}\|}$

- **Form** matrices **G**, **E** and **A = EG^T**
- **Let**

$${}^w\mathbf{h} = \mathbf{A}^s \mathbf{h}$$

- **Let**

$${}^w\mathbf{h} = [h_0 \quad h_1 \quad h_2]$$

$${}^w\mathbf{b} = \left[\sqrt{h_0^2 + h_1^2} \quad 0 \quad h_2 \right]$$

- **Let**

$${}^w\mathbf{b} = [b_0 \quad 0 \quad b_1]$$

- if $a == \text{TRUE}$ then

$$\mathbf{f}_a = \mathbf{A}^w \mathbf{g} - {}^s \mathbf{a}_{i+1}$$

$$\mathbf{f}_h = \mathbf{A}^w \mathbf{b} - {}^s \mathbf{h}_{i+1}$$

$$\mathbf{f} = \mathbf{f}_a + \mathbf{f}_h$$

else

$$\mathbf{f}_h = \mathbf{A}^w \mathbf{b} - {}^s \mathbf{h}_{i+1}$$

$$\mathbf{f} = \mathbf{f}_h$$

- **Let**

$$\mathbf{K} = \begin{bmatrix} -e_2 & e_3 & -e_0 & e_1 \\ e_1 & e_0 & e_3 & e_2 \\ 0 & -2e_1 & -2e_2 & 0 \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} 0 & 0 & -2e_2 & -2e_3 \\ -e_3 & e_2 & e_1 & -e_0 \\ e_2 & e_3 & e_0 & e_1 \end{bmatrix} \quad (1)$$

- **if $a == \text{TRUE}$ then**

$$\mathbf{f}^T \mathbf{J} = 2 (\mathbf{f}_a^T + b_1 \mathbf{f}_h^T) \mathbf{K} + 2b_0 \mathbf{f}_a^T \mathbf{L} \quad (2)$$

else

$$\mathbf{f}^T \mathbf{J} = 2b_1 \mathbf{f}_h^T \mathbf{K} + 2b_0 \mathbf{f}_a^T \mathbf{L} \quad (3)$$

- **Compute**

$$\Delta \mathbf{q} = \frac{\mathbf{J}^T \mathbf{f}}{\|\mathbf{J}^T \mathbf{f}\|}$$

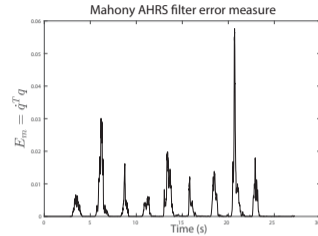
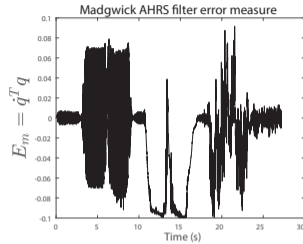
- **Compute:** $\dot{\mathbf{q}}_{\omega_{i+1}} = \frac{1}{2} \mathbf{G}^T \mathbf{s}_{\omega_{i+1}}$
- **Compute:** $\dot{\mathbf{q}}_{est_{i+1}} = (\dot{\mathbf{q}}_{\omega_{i+1}} - \beta \Delta \mathbf{q})$
- **Compute:** $\mathbf{q}_{i+1} = \mathbf{q}_i + \dot{\mathbf{q}}_{est_{i+1}} \Delta t$
- **Normalize** \mathbf{q}_{i+1}

Proposed Refinement

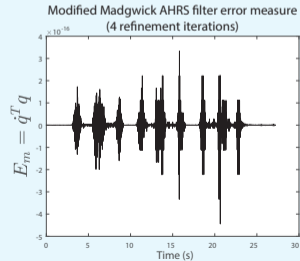
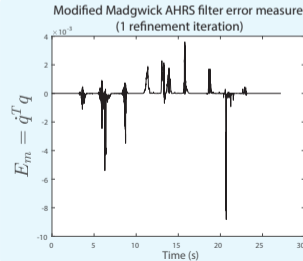
for $i = 1 : n$

- **Compute** E_m
- **Form** \mathbf{G} with updated quaternion value \mathbf{q}_{i+1}
- **Update** $\dot{\mathbf{q}}_{\omega_{i+1}} = \frac{1}{2} \mathbf{G}^T \mathbf{s} \omega_{i+1}$
- **Compute** correction term $\Delta \mathbf{q}_{\text{ref}} = -2 \dot{\mathbf{q}}_{\omega_{i+1}} E_m^3$
- **Update** $\mathbf{q}_{i+1} \leftarrow \frac{\mathbf{q}_{i+1} - \Delta \mathbf{q}_{\text{ref}}}{\|\mathbf{q}_{i+1} - \Delta \mathbf{q}_{\text{ref}}\|}$

Results

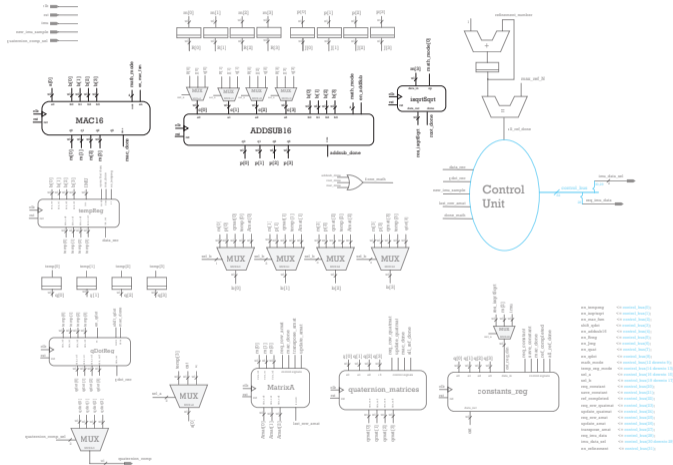


This investigation

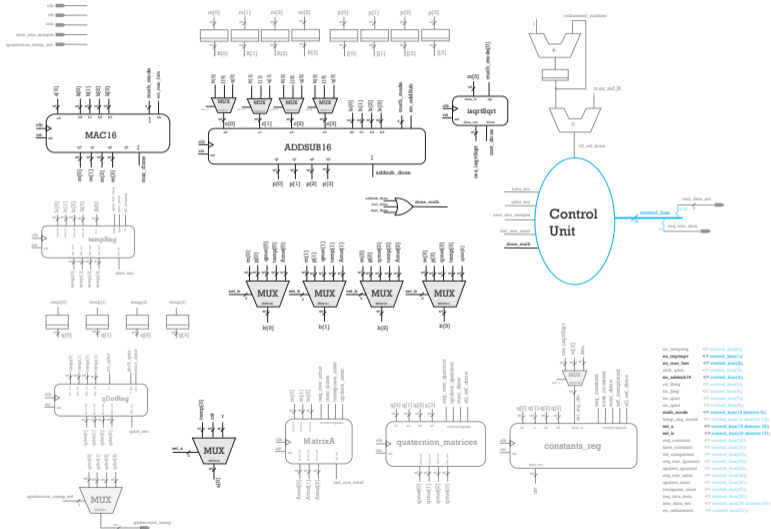


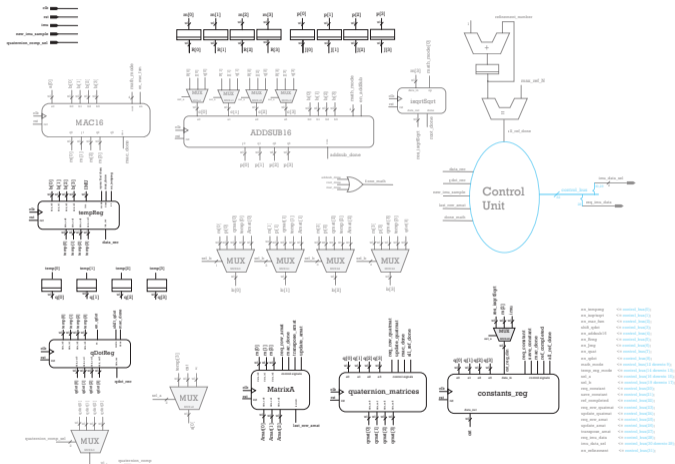
Comparison with other algorithms

	Madgwick	Mahony	Matlab IMU	Proposed Algorithm No Refinement Step	Proposed Algorithm 1 Refinement Step	Proposed Algorithm 4 Refinement Steps
Execution Time [s]	0.113	0.112	2.09	0.063	0.100	0.102
Max. Error $ E_m $	0.0875	0.0576	2.2204e-16	0.0875	0.0116	3.3307e-16



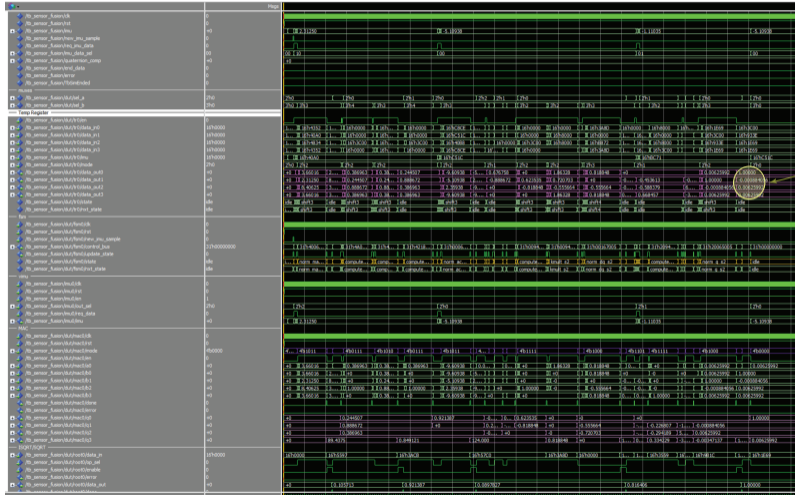
- MAC16 - vector scalar product, vector by scalar operations and matrix products.
- ADDSUB16 - vectors additions.
- isqrtSqrt - computation of both ISQRT and SQRT.





- tempReg - register to hold temporary data.
- MatrixA - stores the content of matrix **A**.
- quaternion_matrices - stores quaternion dependent matrices (e.g. **E,L,K**).
- constants_reg - stores algorithm constants.

Simulation Results



Synthesis Results

Table: Resource Utilization

Logic Utilizations (in ALMs)	2392/29080 (8%)
Total Registers	2665
Total Block Memories	1865/4567040 (< 1%)
Total DSP Blocks	5/130 (3%)

Target FPGA: 5CEFA5F23C7 (Cyclone V Family)

Timing Analysis

Table: Max. Frequency

Model	Max. Frequency [MHz]
Slow 1100mV 85C	62.41
Slow 1100mV 0C	62.63
Fast 1100mV 85C	114.80
Fast 1100mV 0C	128.35

A \sqrt{x} and $1/\sqrt{x}$ Chebyshev polynomial approximation

Let us denote with $x \in \mathbb{R}^+$ a number on which to apply the operations of \sqrt{x} or $\frac{1}{\sqrt{x}}$. We assume that this number is expressed according to IEEE754 standard:

$$x = (-1)^s 2^{e_u} \cdot m \quad (4)$$

where $m \in [1, 2)$ is the mantissa and e_u is the *unbiased* exponent ($e = e_u + \text{bias}$). The following four cases can be distinguished:

- **isqrt even exponent**

$$\frac{1}{\sqrt{x}} = 2^{-\frac{e_U}{2}} \cdot \frac{1}{\sqrt{m}} \quad (5)$$

- **isqrt odd exponent**

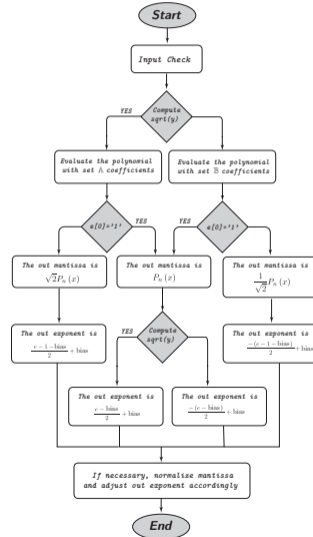
$$\frac{1}{\sqrt{x}} = 2^{-\frac{e_U+1}{2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{m}} \quad (6)$$

- **sqrt even exponent**

$$\sqrt{x} = 2^{\frac{e_U}{2}} \cdot \sqrt{m} \quad (7)$$

- **sqrt odd exponent**

$$\sqrt{x} = 2^{\frac{e_U-1}{2}} \cdot \sqrt{2} \cdot \sqrt{m} \quad (8)$$



The approximation of functions problem

- There is no scientific method of determining which approximating function $F_n(x)$ leads to the most efficient approximation of a function $f(x)$.
- Experience and intuition play an important role in choosing the approximating function

Chebyshev .vs. Least-Square

	Least-Squares (Gauss)	Min-Max (Chebyshev)
Norm to be Minimized	$\sum f(x) - F_n(x) ^2$	$\max_{x_0 \leq x \leq x_{n+1}} f(x) - F_n(x) $
Max. error Estimation	✗	✓
Points of max. error	✗	✓
Equioscillation of error func.	✗	✓

Chebyshev Optimality Criterion

Given a system of powers of x , $(1, x, x^2, x^3, \dots, x^n)$, the Chebyshev min-max polynomial approximation of degree n of a continuous function $f(x)$, over an interval $[x_0, x_{n+1}]$, is the function

$$F_n(x) = p_0 + p_1x + p_2x^2 + \dots + p_nx^n \quad (9)$$

such that $\varepsilon(x)$ is minimized, where

$$\varepsilon(x) = \max_{x_0 \leq x \leq x_{n+1}} |f(x) - F_n(x)| \quad (10)$$

It can be shown that $F_n(x)$ is the Chebyshev approximation of a differentiable function $f(x)$ on $[x_0, x_{n+1}]$ if and only if there exists a set of points

$$x_0 \leq x_1 \leq x_2 \leq \dots \leq x_{n+1}$$

such that:

$$\varepsilon(x_i) = (-1)^i L \quad (i = 0, 1, \dots, n+1) \quad (11a)$$

$$\left. \frac{d\varepsilon(x)}{dx} \right|_{x=x_j} = 0 \quad (j = 1, 2, \dots, n) \quad (11b)$$

where L denotes the maximum value of ε in $[x_0, x_{n+1}]$.

The approximating polynomial for $1/\sqrt{x}$ and \sqrt{x} is obtained by solving the following non linear system:

$$(-1)^{j+1} L + \sum_{i=0}^n p_i x_j^i - f(x) = 0 \quad (j = 0, 1, 2, \dots, n+1) \quad (12a)$$

$$\sum_{i=1}^n i p_i x_k^{i-1} - \left. \frac{df(x)}{dx} \right|_{x=x_k} = 0 \quad (k = 1, 2, \dots, n) \quad (12b)$$

whose solution yields the unknowns $L, x_1, x_2, \dots, x_n, p_0, p_1, \dots, p_n$. Therefore, not only the coefficients of the optimal approximating polynomial are computed, but also the value of maximum error L , as well as the abscissae where maxima and minima of $\varepsilon(x)$ occur.

Approximation Accuracy Result

$1/\sqrt{x}$			\sqrt{x}		
n	N	MRE (this inv.)	n	N	MRE (this inv.)
2	1	3.8335e-03	2	1	7.6384e-04
2	2	7.1823e-04	2	2	1.4346e-04
2	4	1.1463e-04	2	4	2.2916e-05
2	8	1.6430e-05	2	8	3.2855e-06
2	16	2.2090e-06	2	16	4.4179e-07
2	32	2.8674e-07	2	32	5.7347e-08
2	64	3.6537e-08	2	64	7.3073e-09
3	1	5.7648e-04	3	1	8.2059e-05
3	2	6.3523e-05	3	2	9.0636e-06
3	4	5.5903e-06	3	4	7.9832e-07
3	8	4.2321e-07	3	8	6.0452e-08
3	16	2.9293e-08	3	16	4.1847e-09
3	32	1.9301e-09	3	32	2.7573e-10
4	1	8.9120e-05	4	1	9.8694e-06
4	2	5.7777e-06	4	2	6.4124e-07
4	4	2.8042e-07	4	4	3.1147e-08
4	8	1.1213e-08	4	8	1.2457e-09

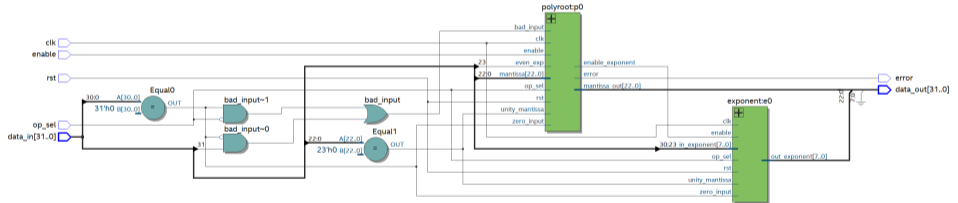
ULP and relative error

The relative error corresponding to $\frac{1}{2}$ ulp for a binary FP format with a precision of p bits is defined by

$$2^{-(p+1)} < \frac{1}{2} \text{ulp} \leq 2^{-p} \quad (13)$$

- p for IEEE 754 half precision is 10
- p for IEEE 754 single precision is 23
- p for IEEE 754 double precision is 51

Proposed FPGA Architecture



Computation of output Mantissa

Let x be the input mantissa, the output mantissa is computed by evaluating the polynomial $P_2(x)$.

Four cases can be distinguished:

- **isqrt even exponent**

$$P_2(x) = (p_{20}x + p_{10})x + p_{00} \quad (14)$$

- **isqrt odd exponent**

$$P_2(x) = (p_{20}x + p_{10})\frac{x}{\sqrt{2}} + p_{01} \quad (15)$$

$$\text{where } p_{01} = \frac{p_{00}}{\sqrt{2}}$$

- **sqrt even exponent**

$$P_2(x) = (p_{22}x + p_{12})x + p_{02} \quad (16)$$

- **sqrt odd exponent**

$$P_2(x) = 2 \left[(p_{22}x + p_{12}) \frac{x}{\sqrt{2}} + p_{03} \right] \quad (17)$$

$$\text{where } p_{03} = \frac{p_{02}}{\sqrt{2}}$$

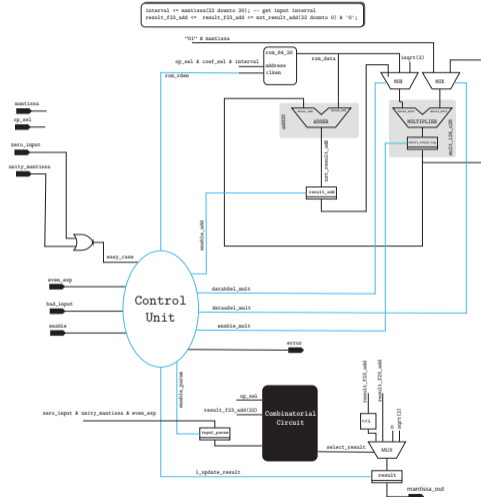
Approximating Polynomial Evaluation

- **Hardware Resources:**

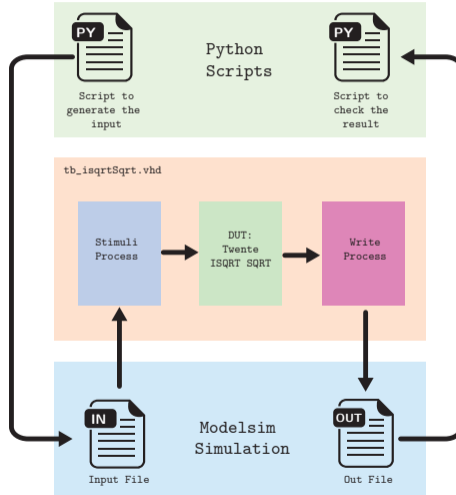
- 1 × Adder
- 1 × Multiplier
- 1 × ROM
- Registers to hold $\frac{1}{\sqrt{2}}$ constant, multiplication/addition results and the result mantissa.

- **Operation Scheduling** $P_2(x) = (p_{22}x + p_{12})kx + p_{02}$

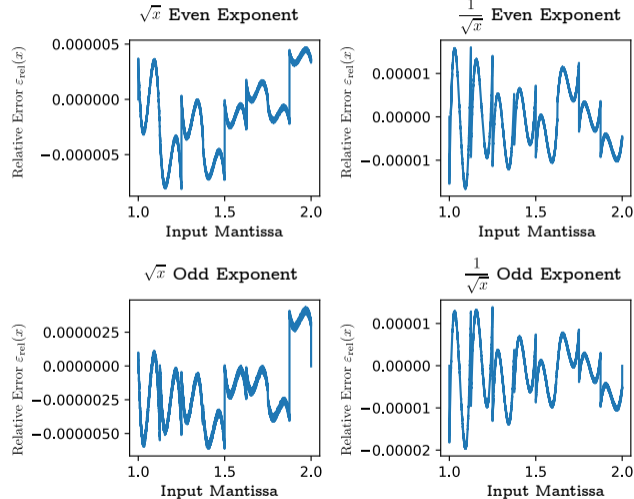
	0	1	2	3
Multiplier	$m \leftarrow p_2x$	$m \leftarrow x \frac{1}{\sqrt{2}}$ or $m \leftarrow x$	$m \leftarrow ma$	
Adder		$a \leftarrow m + p_1$		$r \leftarrow m + p_0$



Simulation Strategy

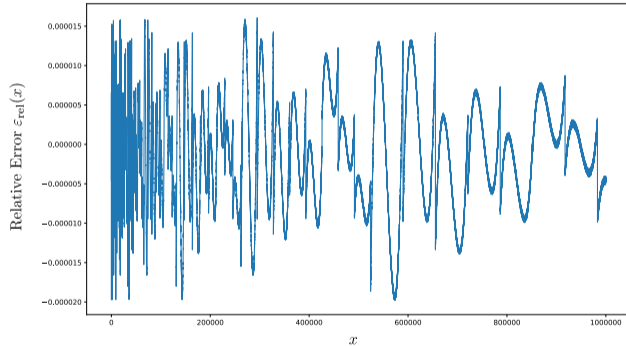


Simulation Results: Mantissa

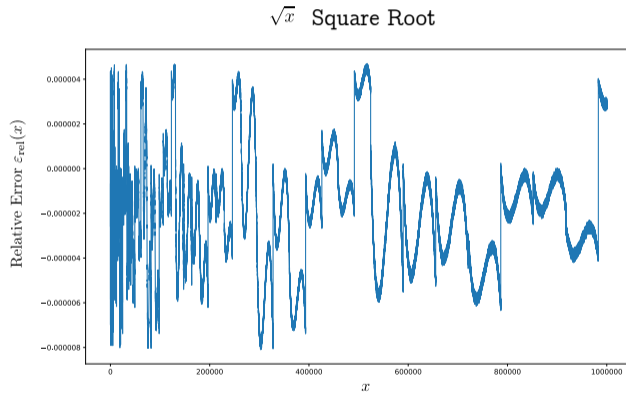


Simulation Results: Inverse Square Root

$$\frac{1}{\sqrt{x}} \text{ Inverse Square Root}$$



Simulation Results: Square Root



Synthesis Results

	Proposed	ALTFP_INV_SQRT	ALTFP_SQRT
Logic Utilizations (in ALMs)	66/9430 (< 1%)	278/9430 (3%)	245/9430 (3%)
Total Registers	60	925	542
Total Block Memories	1280/1802240 (<1%)	565/1802240 (<1%)	143/1802240
Total DSP Blocks	1/25 (4%)	6/25 (24%)	0/25 (0%)
Latency in clk cycles	7	26	16

Target FPGA: 5CEFA2F23I7 (Cyclone V Family)

Timing Analysis

Table: Max. Frequency

Model	Max. Frequency [MHz]
Slow 1100mV 100C	127.28
Slow 1100mV -40C	123.87
Fast 1100mV 100C	230.47
Fast 1100mV -40C	260.28

Conclusions

This work tackled the problem of estimating the spatial orientation of a single 9-DoF IMU platform. The following goals have been achieved:

- Functional outlines of a novel device for bikers comfort vibration monitoring and road condition assessment.

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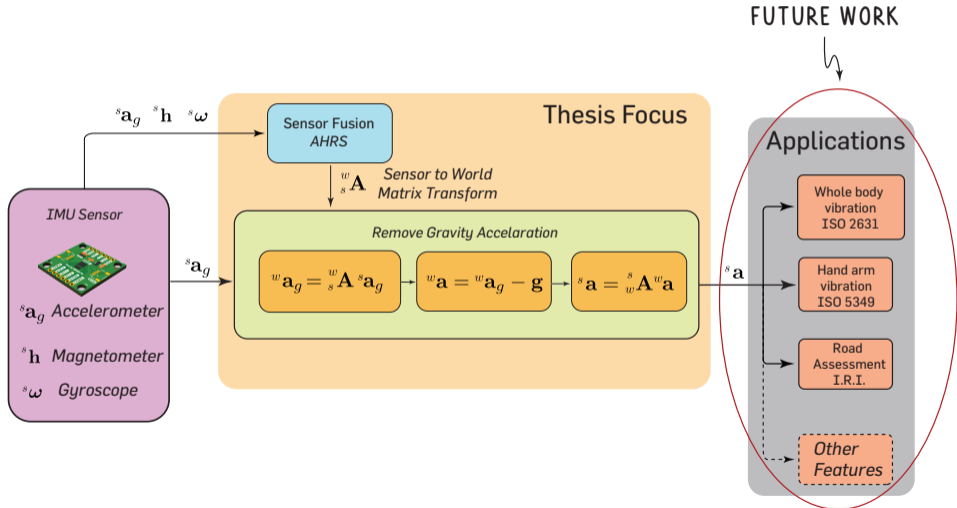
- Functional outlines of a novel device for bikers comfort vibration monitoring and road condition assessment.
- Improvement of the Madgwick sensor fusion algorithm both in speed and accuracy.
- A custom architecture to execute the proposed sensor fusion algorithm was implemented.

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This work tackled the problem of estimating the spatial orientation of a single 9-DoF IMU platform. The following goals have been achieved:

- Functional outlines of a novel device for bikers comfort vibration monitoring and road condition assessment.
- Improvement of the Madgwick sensor fusion algorithm both in speed and accuracy.
- A custom architecture to execute the proposed sensor fusion algorithm was implemented.
- Development of an original method and architecture to compute, with shared resources, both $\frac{1}{\sqrt{x}}$ and \sqrt{x} with a polynomial approximation.

Future Work



Thank you for your attention.